

Exercise 45

Find the derivative. Simplify where possible.

$$y = \coth^{-1}(\sec x)$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned} y' &= \frac{d}{dx}[\coth^{-1}(\sec x)] \\ &= \frac{1}{1 - (\sec x)^2} \cdot \frac{d}{dx}(\sec x) \\ &= \frac{1}{1 - \sec^2 x} \cdot (\sec x \tan x) \\ &= \frac{1}{1 - (\tan^2 x + 1)} \cdot (\sec x \tan x) \\ &= -\frac{1}{\tan^2 x} \cdot (\sec x \tan x) \\ &= -\frac{\sec x}{\tan x} \\ &= -\frac{1}{\cos x \left(\frac{\sin x}{\cos x}\right)} \\ &= -\frac{1}{\sin x} \\ &= -\csc x \end{aligned}$$